Name————————————————, ID ————————————

Exam II MTH 418, Spring 2016

Ayman Badawi

QUESTION 1. (i) Let D be a connected planar graph and $cl(D)$ be the closure graph of D. I claim that $cl(D)$ need not be a planar. Give me an example to support my claim.

(ii) We know that K_5 is not a planar. Find the minimum number of edges that you need to remove from K_5 so that the remaining graph is a planar (you are not allowed to remove vertices, only remove edges).

(iii) We know that W_8 is a planar. Convince me CLEARLY that $\overline{W_8}$ (the complement graph of W_8) is not a planar.

(iv) Find a maximum matching set for Q_3. Does Q_3 have a perfect matching set?

(v) For each $n \geq 3$, convince me that there is a connected graph, say H, that is Hamiltonian but neither Eulerian nor Eulerian trail and $\chi'(H) = n$.

(vi) Give me an example of a connected Eulerian trail, say H, that is neither Hamiltonian nor Eulerian nor critical such that $\chi(H) = 3$ and ONLY one vertex in the trail is visited twice. .

(vii) Let $H = K_3$ with vertex set $\{v_1, v_2, v_3\}$, $D = K_3$ with vertex set $\{w_1, w_2, w_3\}$. Consider the product graph $F = H \times D$. Find $\chi'(F)$. Show that F is not planar [hint: construct a subgraph of F that is a subdivision of $K_{3,3}$]. Is F Eulerian? explain.

(viii) Let F be a connected graph such that $\chi(F) = \chi'(F) + 1$. Find all possibilities of F. Explain!

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com